



# Geometry

## Curriculum Sample

What future do you envision for your student? Straight-A student? Valedictorian? Stanford or Harvard graduate? Successful doctor or engineer?

### Our strong math curriculum can help your student reach his or her goals!

A Grade Ahead's math program introduces and builds upon math concepts every week to strengthen your child's math capabilities over time.

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We make it easy to implement at home! Here's how it works:

- 1. Learn a lesson:** New topics are introduced each week. *(Older students can teach themselves with our easy-to-understand lesson. Younger students may need to be assisted by a parent.)*
- 2. Complete three days of homework exercises.** *(Select the time and place to complete the homework around your schedule.)*
- 3. Also complete four days of numerical drills** to practice speed and accuracy.
- 4. Check your student's success** with the answers provided.
- 5. Enter scores in our Parent Portal** to follow your student's achievements.

### Want to see how A Grade Ahead works first-hand?

We have attached an entire lesson and one day's worth of homework for you to print out and try.



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# Lesson Booklet Sample

## Geometry

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**Geometry Terms, Algebraic Properties, and Addition Postulates**



**Teaching Tip:** Students should be familiar with lines, planes, and angles. Sections A and B (with the exception of constructions) can be reviewed quickly to allow for more focus on Sections C and D. Algebraic Properties can be easily referred back to during homework.

**Student Goals:**

- ✓ I will be able to identify and explain examples of key geometric terms.
- ✓ I will be able to identify midpoints and bisectors.
- ✓ I will be able to construct segment bisectors and duplicate segments.
- ✓ I will be able to find missing values using the addition postulates.
- ✓ I will be able to identify the properties.

**A. Geometry Introduction and Term Review**

Geometry helps us understand the math behind what surrounds us. We use geometry to discover more about and make comparisons between two- and three-dimensional shapes, planes, lines, and points, and we use that information to make even more discoveries.

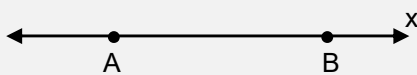

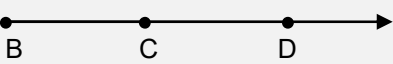
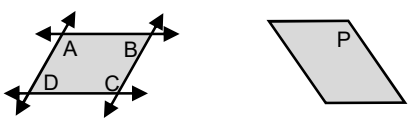
In geometry, students will study facts that have already been proven true as well as the methods used to prove them. This process relies heavily on definitions, interpreting figures, and eliminating assumptions. One of the main rules of geometry is not to assume.

Congruence

When comparing two or more objects in geometry, we use the term **congruent** to describe objects that have the same size and shape but are still separate objects from each other. We use the symbol  $\cong$  to indicate that objects are congruent. Tick marks indicate congruent sides, and curves mark congruent

angles:  

Review Terms Day 1 Q1-8

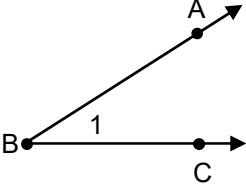
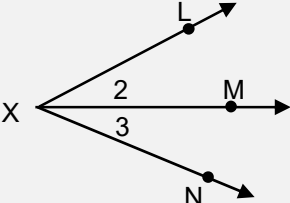
Term	Definition	Example Image	Name(s)
Point	A place in space that can be described by location with no length or width	• A	Capital Letter Ex. Point A
Line	A straight path made of infinite points with no endpoints: It goes forever in both directions.		Line x Line AB $\overleftrightarrow{AB}$
Line Segment	A part of a line consisting of two endpoints and all of the points between them		Line segment $\overline{AD}$ AD
Ray	A part of a line with one endpoint and extending indefinitely in the other direction		Name by endpoint first. Ray BD $\overrightarrow{BD}$
Plane	A flat surface that extends in two directions indefinitely with no height or thickness		Plane ABCD Plane P



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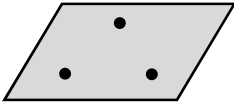
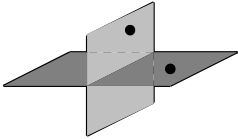
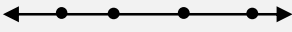
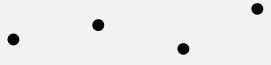
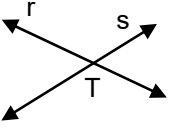
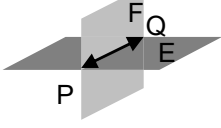
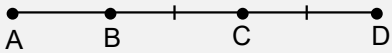
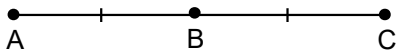
Term	Definition	Example Image	Name(s)
Angle	Two rays with the same endpoint (vertex): Each ray is called a side.		1 angle: $\angle 1$ , $\angle B$ , or $\angle ABC$
Adjacent Angles	Two angles that share a vertex and have a common side		3 different angles: $\angle X$ or $\angle LXN$ $\angle 2$ or $\angle LXM$ $\angle 3$ or $\angle MXN$

**B. Lines and Planes**

Euclidean Space

Day 1 Q1-8

Euclidean Space is the set of all points in three dimensions (x, y, z) or the points that we can see. All topics in this class will be based on this concept of Euclidean Geometry unless otherwise stated.

Term	Definition	Example Images
Coplanar Points	Points that lie in the same plane	  <p>Coplanar points                      Non-coplanar points</p>
Collinear Points	Points that lie in the same line (collinear points are also coplanar.)	  <p>Collinear points                      Non-collinear points</p>
Intersection	When a line or plane meets, cuts across, or overlaps another line or plane	  <p>Line r and line s intersect at T.                      Plane E and plane F intersect at <math>\overline{PQ}</math>.</p>
Congruent Segments	Line segments that are equal in length ( $\overline{BC} \cong \overline{CD}$ ). Tick marks are used to show congruence in a figure.	 <p>Line Segment <math>\overline{BC} \cong</math> Line Segment <math>\overline{CD}</math></p>
Midpoint of a Segment	Divides a segment into two congruent segments	 <p><math>\overline{AB} \cong \overline{BC}</math></p>



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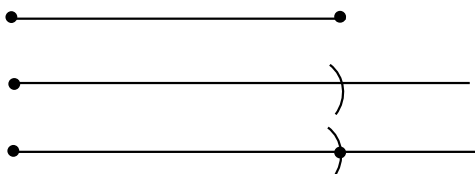



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Duplicating a Line Segment

Day 1 Q37

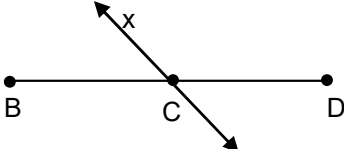
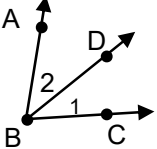
1. If a line segment is not provided, draw one using the straight edge.
2. Draw a second, longer line segment using the straight edge.
3. Next, place the metal or plastic point of the compass on one end of the line segment (the point should be the part that does not have a pencil).
4. Adjust the opening of the compass so that it is equal to the length of the line segment.
5. Without changing the spacing of the compass, place the metal or plastic point of the compass on the left end of the second line segment and draw a small curve across the line.
6. Make a point where the curve crosses the line. The distance from the end point to the created point is the duplicate of the original line segment.

**Teaching Tip:** This construction and the one to follow can both be demonstrated on the board for students to see while students construct their own. Make sure they follow you at each step, to help them remember.

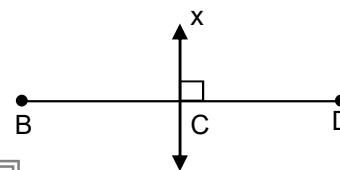
**C. Bisectors**

A bisector is a line, segment, ray, or plane that cuts another figure into two equal parts. The two most common types of bisectors are segment bisectors and angle bisectors (they are named for the figure that they bisect).

Segment Bisector	Angle Bisector
<p>A segment bisector crosses through the midpoint of a segment, splitting that segment into two equal parts.</p> <p>In this example, line <math>x</math> is a bisector of <math>\overline{BD}</math>; therefore, <math>\overline{BC} \cong \overline{CD}</math>.</p> 	<p>An angle bisector passes through the vertex of an angle and divides that angle into two congruent smaller angles.</p> <p>In this example, ray <math>BD</math> is a bisector of <math>\angle ABC</math>; therefore <math>\angle 1 \cong \angle 2</math>.</p> 

Perpendicular Bisector

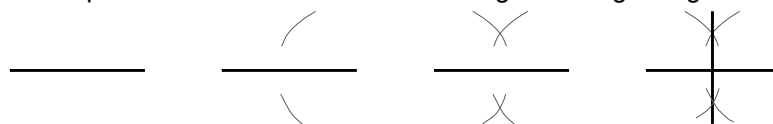
A **perpendicular bisector** is a segment bisector that passes through the midpoint of a segment at a right angle ( $90^\circ$ ). In the figure to the right, line  $x$  is a perpendicular bisector of  $\overline{BD}$ ; therefore,  $\overline{BC} \cong \overline{CD}$ .



Day 1 Q38

Constructing a Perpendicular Bisector for a Line Segment

1. If a line segment is not provided, draw one using the straight edge.
2. Next, place the metal or plastic point of the compass on one end of the line segment (the point should be the part that does not have a pencil).
3. Adjust the opening of the compass so that it is more than half of the line segment.
4. Rotate the compass to draw a small curve above and below the line segment.
5. Without changing the spacing of the compass, move it so that the point is resting on the opposite end of the line segment.
6. Rotate the compass to draw a small curve above and below the line segment (these curves should cross the curves drawn in step 4).
7. Connect the points where the curves cross using the straight edge.



**D. Postulates & Properties**

Postulates and properties have two purposes: they work as explanations to help learn the concepts, and they also work as justifications to help prove new concepts.

Students will be able to recognize new justifications by the dark box that they are written in (such as the box the "Segment Addition POST" is defined in below).

Postulates

Day 1 Q35 & 36

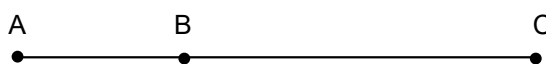
Postulates, also known as axioms, are statements that are accepted as true without needing proof. The abbreviation for a postulate is POST.



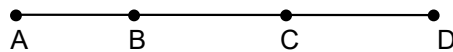
**Teaching Tip:** The Segment and Angle Addition Postulates can both be remembered by thinking about them as the parts added to make a whole and emphasizing that the name of each postulate summarizes its purpose.

**Segment Addition POST**

If B is a point on the line segment  $\overline{AC}$ , then  $m\overline{AB} + m\overline{BC} = m\overline{AC}$ .



**Example:** For the line segment given below, write two equations that equal  $m\overline{AD}$  using the Segment Addition Postulate.



$$m\overline{AB} + m\overline{BD} = m\overline{AD}$$

$$m\overline{AC} + m\overline{CD} = m\overline{AD}$$



**Note:** Using *m* before an angle or segment indicates measure:

$$m\overline{AB} = \text{measure of } \overline{AB}$$

$$m\angle ABC = \text{measure of } \angle ABC$$

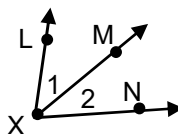
Use *m* when writing an equation with an equal sign, not a congruence sign:

$$m\overline{AB} = 7$$

$\overline{AB} \cong 7$  (Note that some teachers may not accept congruence sign with a number, but in HS, it is accepted.)

**Angle Addition POST**

If ray  $\overrightarrow{XM}$  lies in the interior of  $\angle LXN$ , then  $m\angle LXM + m\angle MXN = m\angle LXN$



**Example:** Solve for  $m\angle 1$  and  $m\angle 2$  given that the following is true:  
 $m\angle ABC = x + 45^\circ$ ;  $m\angle 1 = x + 5$ ;  $m\angle 2 = x - 2$

$$m\angle ABC = m\angle 1 + m\angle 2$$

$$x + 45 = x + 5 + x - 2$$

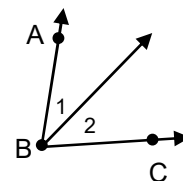
$$x + 45 = 2x + 3$$

$$x = 42$$

Angle Addition Postulate

$$m\angle 1 = 42 + 5 = 47^\circ$$

$$m\angle 2 = 42 - 2 = 40^\circ$$



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Properties of Equality

Day 1 Q16-26

Review Properties	Definition	Example
Addition Property	If $a = b$ , then, $a + c = b + c$ .	Solve for $x$ : $x - 3 = 15$ $x - 3 + 3 = 15 + 3$ $x = 18$ Addition PTY
Subtraction Property	If $a = b$ , then, $a - c = b - c$ .	Solve for $m \angle A$ : $m \angle A + m \angle B = 98^\circ$ $m \angle A + m \angle B - m \angle B = 98^\circ - \angle B$ $m \angle A = 98^\circ - m \angle B$ Subtraction PTY
Multiplication Property	If $a = b$ , then, $ac = bc$ .	Solve for $x$ : $\frac{x}{5} = 2$ $5 \times \frac{x}{5} = 2 \times 5$ $x = 10$ Multiplication PTY
Division Property	If $a = b$ and $c \neq 0$ , then, $\frac{a}{c} = \frac{b}{c}$ .	Solve for $a$ : $a \times 12 = 48$ $a \times 12 \div 12 = 48 \div 12$ $a = 4$ Division PTY
Distributive Property	$a(b + c) = ab + ac$ .	$3(x + 2) = 3x + 6$ $(3)(x) + (3)(2) = 3x + 6$ $3x + 6 = 3x + 6$ Distributive PTY

**The Substitution Property**

If  $a = x$ , and  $a + b = c$ , then,  $x + b = c$ .

The substitution property is similar to those of Addition, Subtraction, Multiplication, and Division, except it allows us to replace part of an original statement with an equivalent term and the statement will stay true.



**Example:**  $m \overline{AB} = m \overline{CD}$ . If  $m \overline{AB} + 6 = 8$ , then, what is  $m \overline{CD}$ ?

$$m \overline{AB} + 6 = 8$$

$$m \overline{CD} + 6 = 8 \quad \text{Substitution PTY}$$

$$m \overline{CD} = 2 \quad \text{Subtraction PTY}$$



*Note: The abbreviation for a property is PTY.*

**The Transitive Property**

If  $a = b$ , and  $b = c$ , then  $a = c$ .

The transitive property allows us to take a previously discovered fact and incorporate it to make a later connection. It is *always at least a 3-step process*.



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**Example:**  $m\overline{AB} = m\overline{CD}$ , and  $m\overline{DE} = 2$ . If  $m\overline{AB} + 6 = 8$ , then, what how does  $m\overline{CD}$  relate to  $m\overline{DE}$ ?

$$m\overline{AB} + 6 = 8$$

$$m\overline{CD} + 6 = 8 \quad \text{Substitution PTY}$$

$$m\overline{CD} = 2 \quad \text{Subtraction PTY}$$

$$m\overline{DE} = 2$$

$$m\overline{CD} = m\overline{DE} \quad \text{Transitive PTY}$$

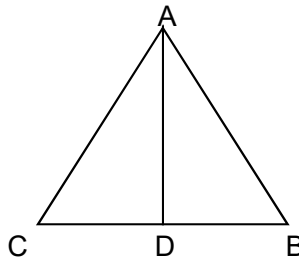
### The Reflexive Property

$$a = a$$

The reflexive property allows us to state that something is equal to itself. This property is most commonly used when 2 objects share a side, or a side needs to be referenced at a later point.



**Example:** How can you prove that  $\triangle ABD$  and  $\triangle ACD$  have at least one congruent side?



Since  $\triangle ABD$  and  $\triangle ACD$  both share side  $AD$  and we are able to state that  $AD \cong AD$  since anything is congruent to itself. Therefore, by the reflexive property,  $\triangle ABD$  and  $\triangle ACD$  have at least one congruent side.

### The Symmetric Property

$$\text{If } a = b, \text{ then } b = a.$$

The symmetric property tells us that two items are equal to each other even when given in the opposite order. This is the property that allows us to flip an equation to put the variable on the left.



**Example:** Solve  $2x + 5 = 3x - 4$  for  $x$ .

$$2x + 5 = 3x - 4$$

$$5 = x - 4 \quad \text{Subtraction Property}$$

$$9 = x \quad \text{Addition Property}$$

$$x = 9 \quad \text{Symmetric Property}$$



*Note: Of these four new properties, the Substitution Property and Transitive Property will be most commonly used in high school geometry. It is important for students to learn the difference between the two in order to tell which is being applied in problems.*



Date: \_\_\_\_\_

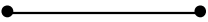
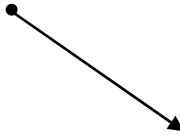

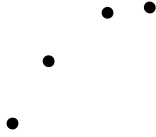
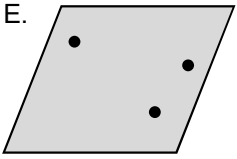
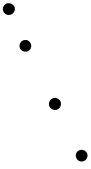

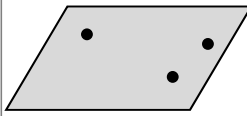
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End Time: \_\_\_\_\_

Score: \_\_\_\_/40

**You may use a calculator unless otherwise indicated. Figures are not drawn to scale.**

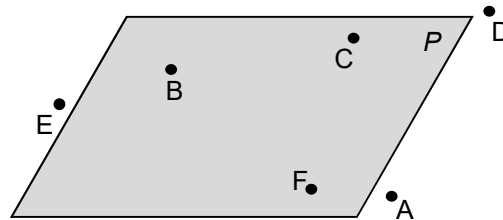
Match the image with the correct term.

<p>A.</p> 	<p>B.</p> 	<p>C.</p> 	<p>D.</p> 
<p>E.</p> 	<p>F.</p> 	<p>G.</p> 	<p>H.</p> 

- |                        |       |                         |       |
|------------------------|-------|-------------------------|-------|
| 1. Collinear Points    | _____ | 2. Line Segment         | _____ |
| 3. Non-coplanar Points | _____ | 4. Point                | _____ |
| 5. Ray                 | _____ | 6. Non-collinear Points | _____ |
| 7. Line                | _____ | 8. Coplanar Points      | _____ |

Use your knowledge of lines and planes to answer the following questions.

9. How many points are necessary to determine a line? \_\_\_\_\_
10. How many points are needed to be non-collinear? \_\_\_\_\_
11. How many points are necessary to determine a plane? \_\_\_\_\_



12. What is the name of the plane above? \_\_\_\_\_
13. List the points that are coplanar. \_\_\_\_\_
14. List the points that are non-coplanar with the given plane. \_\_\_\_\_
15. Name an example of a plane from the room you are in. \_\_\_\_\_



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On each blank below, use your knowledge of the algebraic properties of equality to rewrite the underlined portion of each statement so that it is true for the property given.

16. By the **symmetric property**, we know that if  $a = 7$ , then  $9 = a$ .

\_\_\_\_\_

17. By the **transitive property**, we know that if Line A is congruent to Line B, and Line B is congruent to Line C, then Line A is not congruent to Line C.

\_\_\_\_\_

18. By the **substitution property**, we know that if  $a = x$ , and  $a + 9 = 21$ , then  $x + 21 = 9$ .

\_\_\_\_\_

19. By the **distributive property**, we know that if  $6(x - 5) = 26$ , then  $6x - 5 = 26$ .

\_\_\_\_\_

20. By the **reflexive property**, we know that if Triangle ABC and Triangle DBC share side  $\overline{BC}$ , then  $\overline{BC} \cong \overline{AC}$ .

\_\_\_\_\_

Solve for  $b$ . Then, indicate which property the original equation is defining and which property is being used to solve.

21-23.  $19 - 16 = b - 16$

$b =$  \_\_\_\_\_

Defining: \_\_\_\_\_

Solved by: \_\_\_\_\_

24-26.  $5(3) = 5b$

$b =$  \_\_\_\_\_

Defining: \_\_\_\_\_

Solved by: \_\_\_\_\_

27-29.  $b + 34 = 27 + 34$

$b =$  \_\_\_\_\_

Defining: \_\_\_\_\_

Solved by: \_\_\_\_\_

30-32.  $\frac{7}{11} = \frac{b}{11}$

$b =$  \_\_\_\_\_

Defining: \_\_\_\_\_

Solved by: \_\_\_\_\_



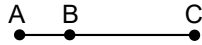
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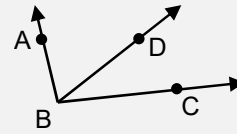
Solve the following using the Segment Addition POST or the Angle Addition POST.

33. Given  $m\overline{AC} = 13$  and  $m\overline{BC} = 8$ ; what is  $m\overline{AB}$ ?



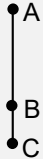
$m\overline{AB} = \underline{\hspace{2cm}}$

34. Given  $m\angle ABD = 43^\circ$ , and  $m\angle CBD = 38^\circ$ , what is  $m\angle ABC$ ?



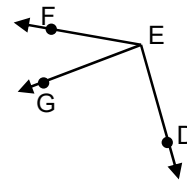
$m\angle ABC = \underline{\hspace{2cm}}$

35. Given  $m\overline{AC} = 18$  and  $m\overline{AB} = \frac{2}{3}\overline{AC}$ , what is  $m\overline{BC}$ ?



$m\overline{BC} = \underline{\hspace{2cm}}$

36. Given  $m\angle DEG = 2x + 5^\circ$ ,  $m\angle FEG = x - 3^\circ$  and  $m\angle DEF = 122^\circ$ ; what is  $m\angle FEG$ ?



$m\angle FEG = \underline{\hspace{2cm}}$

Complete each construction as directed.

37. Duplicate the segment below



38. Construct a Perpendicular Bisector for the segment below.



Circle the step in which this student made an error in duplicating a line segment and constructing a perpendicular bisector. Then, draw what the correct step should have been beneath the images.

39. Duplicating a Line Segment

Step 1:

Step 2:

Step 3:

40. Constructing a Perpendicular Bisector

Step 1:

Step 2:

Step 3:

Step 4:



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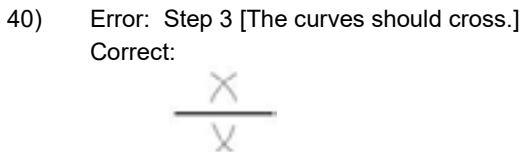
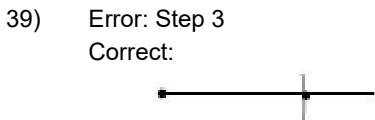
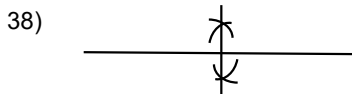
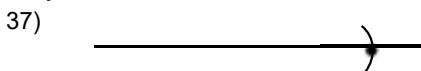
Week: 1 – Day 1

- |                 |                 |
|-----------------|-----------------|
| 1) F            | 2) A            |
| 3) H            | 4) C            |
| 5) B            | 6) D            |
| 7) G            | 8) E            |
| 9) 2            | 10) 3           |
| 11) 3           | 12) Plane P     |
| 13) B, C, and F | 14) A, D, and E |

Answers to question 15 may vary, but must be a flat surface in the room. An example is given.

- |   |                                   |
|---|-----------------------------------|
| 15) The floor [A table, a wall, a book, or any other flat surface in the room could also be correct.]   |                                   |
| 16) $7 = a$   |                                   |
| 17) Line A is congruent to Line C.  |                                   |
| 18) $x + 9 = 21$  |                                   |
| 19) $6x - 30 = 26$ OR $6x - (6)(5) = 26$  |                                   |
| 20) $\overline{BC} \cong \overline{BC}$   |                                   |
| 21) 19  | 22) Subtraction PTY               |
| 23) Addition PTY  | 24) 3                             |
| 25) Multiplication PTY  | 26) Division PTY                  |
| 27) 27  | 28) Addition PTY                  |
| 29) Subtraction PTY   | 30) 7                             |
| 31) Division PTY  | 32) Multiplication PTY            |
| 33) 5 [ $13 - 8 = 5$ ]  | 34) $81^\circ$ [ $43 + 38 = 81$ ] |
| 35) $6 \left[ \frac{2}{3}(18) = 12; 18 - 12 = 6 \right]$  |                                   |
| 36) $37^\circ$ [ $2x + 5 + x - 3 = 122^\circ$ ; $3x + 2 = 122^\circ$ ; $3x = 120^\circ$ ; $x = 40^\circ$ ; $\angle FEG = 40 - 3 = 37^\circ$ ] |                                   |

For questions 37-38, the drawings are not shown to scale, but the marks needed to complete the constructions correctly are shown.





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