

A Grade Ahead's rigorous, year-round math enrichment program is designed to challenge your child to a higher academic standard. Our monthly curriculum includes mathematical concepts that your child will see in school. Your child will learn and apply math concepts to real-world situations through word problems and develop strong critical thinking and analytical skills.

Each week will have an in-depth lesson (which we call Examples), homework, and answers. In these next pages, we offer a closer look at what our Examples, homework, and answers offer as well as a specific example of each.

	Examples - Geometry [Grades 9-10]					
	Æ	Example: $m \overline{AB} = m \overline{DE}$ ? $m \overline{AB} + 6 = 8$ $m \overline{CD} + 6 = 8$ $m \overline{CD} = 2$ $m \overline{DE} = 2$ $m \overline{CD} = m \overline{DE}$	m CD, and m DE = 2. If m AB + 6 = 8,then, what how does m CD relate to Substitution PTY Subtraction PTY Transitive PTY			
	The Refle	xive Property				
	The reflexive property allows us to state that something is equal to itself. This property is most commonly used when 2 objects share a side, or a side needs to be referenced at a later point.					
	Since $\Delta$ ABD and $\Delta$ ACD both share side AD and we are able to state that AD $\approx$ AD since anything is congruent to itself. Therefore, by the reflexive property, $\Delta$ ABD and $\Delta$ ACD have at least one congruent side.					
netric Property 2						
	that two items are equal to each other exercise					

## Examples

To illustrate the topic, examples are provided to you and your child. These examples help demonstrate how to solve the problem or figure out the answer.



Lesson pages are titled "Examples – Geometry [Grades 9–10]," answer pages are titled "Answers – Geometry [Grades 9–10]," and homework pages are simply titled "Geometry [Grades 9–10]."



## Homework

Each week, four days of homework are given to apply concepts from that week's lesson and reinforce the topic.



### Answers

Answers are provided to check your child's homework. Enter the scores into the Parent Portal to track progress and note which areas may need more work.

# Geometry Terms, Algebraic Properties, and Addition Postulates



**Teaching Tip:** Students should be familiar with lines, planes, and angles. Sections A and B (with the exception of constructions) can be reviewed quickly to allow for more focus on Sections C and D. Algebraic Properties can be easily referred back to during homework.

#### A. Geometry Introduction and Term Review

Geometry helps us understand the math behind what surrounds us. We use geometry to discover more about and make comparisons between two- and threedimensional shapes, planes, lines, and points, and we use that information to make even more discoveries.



In geometry, students will study facts that have already been proven true as well as the methods used to prove them. This process relies heavily on definitions, interpreting figures, and eliminating assumptions. One of the main rules of geometry is not to assume.

#### <u>Congruence</u>

When comparing two or more objects in geometry, we use the term **congruent** to describe objects that have the same size and shape but are still separate objects from each other. We use the symbol  $\cong$  to indicate that objects are congruent. Tick marks indicate congruent sides, and curves mark congruent

angles:

<u>Review Terms</u>

Day 1 Q1-8

Term	Definition	Example Image	Name(s)
Point	A place in space that can be described by location with no length or width	• A	Capital Letter Ex. Point A
Line	A straight path made of infinite points with no endpoints: It goes forever in both directions.	A B X	Line x Line AB AB
Line Segment	A part of a line consisting of two endpoints and all of the points between them	A B C D	Line segment AD AD
Ray	A part of a line with one endpoint and extending indefinitely in the other direction	B C D	Name by endpoint first. Ray BD ⊟D
Plane	A flat surface that extends in two directions indefinitely with no height or thickness		Plane ABCD Plane P

#### Week: 1

Term	Definition	Example Image	Name(s)
Angle	Two rays with the same endpoint (vertex): Each ray is called a side.		1 angle: ∠ 1, ∠ B, or ∠ ABC
Adjacent Angles	Two angles that share a vertex and have a common side	X 2 M 3 N	3 different angles: ∠ X or ∠ LXN ∠ 2 or ∠ LXM ∠ 3 or ∠ MXN

#### **B.** Lines and Planes

Euclidean Space

Day 1 Q1-8

Euclidean Space is the set of all points in three dimensions (x, y, z) or the points that we can see. All topics in this class will be based on this concept of Euclidean Geometry unless otherwise stated.

Term Definition		Example Images		
Coplanar Points	Points that lie in the same plane	Coplanar points Non-coplanar points		
Collinear Points	Points that lie in the same line (collinear points are also coplanar.)	Collinear points Non-collinear points		
Intersection	When a line or plane meets, cuts across, or overlaps another line or plane	$\begin{array}{c} r \\ \hline \\ \hline \\ \hline \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$		
Congruent Segments	Line segments that are equal in length ( $\overline{BC} \cong \overline{CD}$ ). Tick marks are used to show congruence in a figure.	$\begin{array}{c c} \bullet & \bullet & \bullet & \bullet \\ A & B & C & D \\ \hline \\ Line Segment BC \cong Line Segment CD \end{array}$		
Midpoint of a Segment	Divides a segment into two congruent segments	$ \begin{array}{c c} \bullet & \bullet & \bullet \\ A & B & C \\ \hline \overline{AB} \cong \overline{BC} \end{array} $		

#### Duplicating a Line Segment

Day 1 Q37

- 1. If a line segment is not provided, draw one using the straight edge.
- 2. Draw a second, longer line segment using the straight edge.
- 3. Next, place the metal or plastic point of the compass on one end of the line segment (the point should be the part that does not have a pencil).
- 4. Adjust the opening of the compass so that it is equal to the length of the line segment.
- 5. Without changing the spacing of the compass, place the metal or plastic point of the compass on the left end of the second line segment and draw a small curve across the line.
- Make a point where the curve crosses the line. The distance from the end point to the created point is the duplicate of the original line segment.



**Teaching Tip:** This construction and the one to follow can both be demonstrated on the board for students to see while students construct their own. Make sure they follow you at each step, to help them remember.

#### C. Bisectors

A bisector is a line, segment, ray, or plane that cuts another figure into two equal parts. The two most common types of bisectors are segment bisectors and angle bisectors (they are named for the figure that they bisect).



#### Perpendicular Bisector

A **perpendicular bisector** is a segment bisector that passes through the midpoint of a segment at a right angle (90°).In the figure to the right, line x

is a perpendicular bisector of BD ; therefore,  $BC \cong CD$  .





#### Constructing a Perpendicular Bisector for a Line Segment

- 1. If a line segment is not provided, draw one using the straight edge.
- 2. Next, place the metal or plastic point of the compass on one end of the line segment (the point should be the part that does not have a pencil).
- 3. Adjust the opening of the compass so that it is more than half of the line segment.
- 4. Rotate the compass to draw a small curve above and below the line segment.
- 5. Without changing the spacing of the compass, move it so that the point is resting on the opposite end of the line segment.
- 6. Rotate the compass to draw a small curve above and below the line segment (these curves should cross the curves drawn in step 4).
- 7. Connect the points where the curves cross using the straight edge.

#### D. Postulates & Properties

Postulates and properties have two purposes: they work as explanations to help learn the concepts, and they also work as justifications to help prove new concepts.

Students will be able to recognize new justifications by the dark box that they are written in (such as the box the "Segment Addition POST" is defined in below).

<u>Postulates</u>

Day 1 Q35 & 36

Postulates, also known as axioms, are statements that are accepted as true without needing proof. The abbreviation for a postulate is POST.  Teaching Tip: The Segment and Angle Addition Postulates can both be remembered by thinking about them as the parts added to make a whole and emphasizing that the name of each postulate summarizes its purpose.



B

**Example:** For the line segment given below,

write two equations that equal m AD using the Segment Addition Postulate.

$$A B C D$$

$$\overline{AB} + m\overline{BD} = m\overline{AD}$$

 $m\overline{AC} + m\overline{CD} = m\overline{AD}$ 



Note: Using **m** before an angle or segment indicates measure:

 $m \overline{AB}$  = measure of  $\overline{AB}$ m  $\angle ABC$  = measure of  $\angle ABC$ 

Use m when writing an equation with an equal sign, not a congruence sign:

 $m\overline{AB} = 7$ 

 $AB \cong 7 \ (\text{Note that some teachers} \\ \text{may not accept congruence sign with} \\ \text{a number, but in HS, it is accepted.} )$ 





**Example:** Solve for  $m \ge 1$  and  $m \ge 2$  given that the following is true:  $m \ge ABC = x + 45^{\circ}$ ;  $m \ge 1 = x + 5$ ;  $m \ge 2 = x - 2$ 

 $m \angle ABC = m \angle 1 + m \angle 2$ x + 45 = x + 5 + x - 2 x + 45 = 2x + 3 x = 42  $m \angle 1 = 42 + 5 = 47^{\circ}$ m  $\angle 2 = 42 - 2 = 40^{\circ}$  Angle Addition Postulate



Properties of Equality Day 1 Q16-26

	·		
<b>Review Properties</b>	Definition	Exan	nple
Addition Property	If a = b, then, a + c = b + c.	Solve for x: x - 3 = 15 x - 3 + 3 = 15 + 3 x = 18	Addition PTY
Subtraction Property	If a = b, then, a – c = b – c.	Solve for $m \angle A$ : $m \angle A + m \angle B = 98^{\circ}$ $m \angle A + m \angle B - m \angle B = 98^{\circ} - m \angle A = 98^{\circ} - m \angle B$	- ∠ B Subtraction PTY
Multiplication Property	If a = b, then, ac = bc.	Solve for x: $\frac{x}{5} = 2$ $5 \times \frac{x}{5} = 2 \times 5$ x = 10	Multiplication PTY
Division Property	If a = b and c $\neq$ 0, then, $\frac{a}{c} = \frac{b}{c}.$	Solve for a: $a \times 12 = 48$ $a \times 12 \div 12 = 48 \div 12$ a = 4	Division PTY
Distributive Property	a(b + c) = ab + ac.	3(x + 2) = 3x + 6(3)(x) + (3)(2) = 3x + 63x + 6 = 3x + 6	Distributive PTY

#### **The Substitution Property**

If a = x, and a + b = c, then, x + b = c.

The substitution property is similar to those of Addition, Subtraction, Multiplication, and Division, except it allows us to replace part of an original statement with an equivalent term and the statement will stay true.



**Example**:  $\overline{MAB} = \overline{MCD}$ . If  $\overline{MAB} + 6 = 8$ , then, what is  $\overline{MCD}$ ?



 $m \frac{AB}{CD} + 6 = 8$  $m \frac{CD}{CD} + 6 = 8$  $m \frac{CD}{CD} = 2$ 

Substitution PTY Subtraction PTY

#### The Transitive Property

If a = b, and b = c, then a = c.

The transitive property allows us to take a previously discovered fact and incorporate it to make a later connection. It is *always at least a 3-step process*.

#### Week: 1

 

 Example: m AB = m CD, and m DE = 2. If m AB + 6 = 8, then, what how does m CD relate to m DE?

 m AB + 6 = 8 m CD + 6 = 8 m CD = 2

 Substitution PTY m CD = 2

 Subtraction PTY m DE = 2 m CD = m DE

 Transitive PTY

The reflexive property allows us to state that something is equal to itself. This property is most commonly used when 2 objects share a side, or a side needs to be referenced at a later point.



**Example**: How can you prove that  $\triangle ABD$  and  $\triangle ACD$  have at least one congruent side?



Since  $\triangle ABD$  and  $\triangle ACD$  both share side AD and we are able to state that AD  $\cong$  AD since anything is congruent to itself. Therefore, by the reflexive property,  $\triangle ABD$  and  $\triangle ACD$  have at least one congruent side.

#### The Symmetric Property

If a = b, then b = a.

The symmetric property tells us that two items are equal to each other even when given in the opposite order. This is the property that allows us to flip an equation to put the variable on the left.



**Example**: Solve 2x + 5 = 3x - 4 for x.

2x + 5 = 3x - 4 5 = x - 4 9 = xx = 9

Subtraction Property Addition Property Symmetric Property



Note: Of these four new properties, the Substitution Property and Transitive Property will be most commonly used in high school geometry. It is important for students to learn the difference between the two in order to tell which is being applied in problems.

te:	Start Time:	End Time:		
u may use a calculator unless c	herwise indicated. Figures are	Score:/4 not drawn to scale.		
Match the image with the corre	t term.			
A. B.	С.	D.		
• • · · · ·	•	•		
		•		
E. / F.	● G. ▲	H. •		
•	•	• •/		
•	•	•		
	•			
1. Collinear Points	2. Line Segmen	2. Line Segment		
3. Non-coplanar Points	4. Point	<ol> <li>Point</li> <li>Non-collinear Points</li> </ol>		
5. Ray	6. Non-collinear			
7. Line	8. Coplanar Poi	ints		
Use your knowledge of lines a	d planes to answer the followin	ng questions.		
9. How many points are necessa	y to determine a line?			
10. How many points are needed	to be non-collinear?			
11. How many points are necess	ry to determine a plane?			
E /	B C P	<b>7</b> <sup>•</sup> D		
	• A			
12. What is the name of the plan	above?			
13. List the points that are coplanar.				
14. List the points that are non-c	planar with the given plane.			

On each blank below, use your knowledge of the algebraic properties of equality to rewrite				
the underlined portion of each statement so that it is true for the property given.				
17. By the <b>transitive property</b> , we know that if congruent to Line C, then <u>Line A is not cong</u>	By the <b>transitive property</b> , we know that if Line A is congruent to Line B, and Line B is congruent to Line C, then Line A is not congruent to Line C.			
18. By the <b>substitution property</b> , we know tha	By the <b>substitution property</b> , we know that if $a = x$ , and $a + 9 = 21$ , then $x + 21 = 9$ .			
19. By the <b>distributive property</b> , we know that	9. By the <b>distributive property</b> , we know that if $6(x - 5) = 26$ , then $6x - 5 = 26$ .			
20. By the <b>reflexive property</b> , we know that if Triangle ABC and Triangle DBC share side $\overline{BC}$ , then $\overline{BC} \cong \overline{AC}$ .				
Solve for b. Then, indicate which property the property is being used to solve.	e original equation is defining and which			
21-23. 19 – 16 = b – 16	24-26. 5(3) = 5b			
b =	b =			
Defining:	Defining:			
Solved by:	Solved by:			
27-29. $b + 34 = 27 + 34$ 30-32. $\frac{7}{11} = \frac{b}{11}$				
b =	b =			
Defining:	Defining:			
Solved by:	Solved by:			



Date:	Sta:	rt Time:	End Time:			
You may use a calculator unle	ss otherwise indica	ted. Figures are not	Score:/49 drawn to scale.			
Solve for the following using the segment and angle addition postulates.						
<pre></pre>	<u>e</u>  e		•			
	B (		D I			
Let $mBD = 24$ , and $mAC = n$	n CD . Solve for each	of the following.	   			
1. mBC =	2. m <del>AB</del> =		3. m CD =			
Let m $\angle AED = 96 + x, m \angle B$	BED = 3x, and m $\angle$ C	$ED = 48^{\circ}$ . Solve for each $e$	ach of the following.			
4. m ∠ BEC =	-		*			
5. m ∠ BED =	_	•	A B C			
6. m ∠ AED =	_					
7. m ∠AEB =	-		E			
8. m ∠AEC =	_		ļ			
Fill in the blank with the pr	etulate that tells vo	u that the equation (	niven is true. If neither			
postulate you have learned	d works, write "none	e". Use the rhombus	given.			
9	$9y + y^2 - 10 = 180^\circ$	A	<u>x+6</u>			
10	_ 9y = 11y - 20		/ ×			
11	$\bar{\mathbf{x}} + \bar{\mathbf{x}} + \bar{\mathbf{x}} \cong \overline{BC}$	x <sup>2</sup> (4y	$(9y)^{\circ}$ (9y) (y <sup>2</sup> - 10) (			
12.	x + 6 = 4x - 3		(11y-20)°			
13	$-9y + 4y + 50 - 180^{\circ}$		$\searrow \neq$			
	_ 0 , 1 , 1 , 1 , 0 = 100	D	4x-3 C			
2	, 					
Draw and label each of the	following on the pla	ane provided.				
		14. Draw a coplanar three points labeled	line segment containing A, B, and C.			
		15. Draw three collin and F that are non-c given.	ear points labeled D, E,			
		16. Draw three non-o labeled G, H, and I.	collinear, coplanar points			
		17. Draw three non-o points labeled J, K, a	collinear, non-coplanar			



Write the logical result and which property of e	equality allows us to make each statement.		
34-35. If a = b and b = c, then a =	· · · · · · · · · · · · · · · · · · ·		
36-37. If b = 5 and 3 + b = c, then 3 +	= c ;		
38-39. If a = b, then b =			
40-41. AB =	;;		
42-43. Duplicate the segment below. Then, consegment using the steps we have learned.	struct a perpendicular bisector of the new		
Fill in each blank with the property used in the final step shown.			
44. $14x - 20 = 2(x - 4)$	45. $7x - 10 = x - 4$		
$\frac{14x-20}{2} = \frac{2(x-4)}{2}$	7x - x - 10 = x - x - 4		
Property:	Property:		
46. $6x - 10 = -4$ 6x - 10 + 10 = -4 + 10	47. $\frac{6x}{7} = 13$ $\frac{6x}{7} \times 7 = 13 \times 7$		
Property:	Property:		
48. f(x) = 3(5x + 3), and x = 5: f(5) = 3(5 × 5 + 3)	49. $9x + 36 = 18x$ 9(x + 4) = 18x		
Property:	Property:		

Week: 1 - Day 3

Date:	Start Time: End Time:					
Do not use a calculator unless other	Score:/39 wise indicated. Figures are not drawn to scale.					
Fill in the blank with the correct term.						
1. A(n)	is a flat surface that extends in 2 directions.					
2. A(n) them.	consists of two endpoints and all of the points between					
3. A(n)	is a place in space with no length or width.					
4. A(n)	is two rays with the same endpoint.					
5. A(n)	is a straight path through at least two points.					
6	_ share a vertex and have a common side.					
7. A(n) direction.	has one endpoint extending indefinitely in the other					
Draw and label each of the follow	ing on the plane provided.					
x	8. Draw a coplanar line segment containing three points labeled A, B, and C on Plane X.					
	9. Draw three collinear points labeled D, E, and F that are non-coplanar to the planes X and Y but pass through both.					
Y	10. Draw three non-collinear, coplanar points labeled G, H, and I on Plane Y.					
	11. Draw three non-collinear, non-coplanar points labeled J, K, and L on neither Plane X nor Plane Y.					
Complete the example for each of	f the following properties of equality					
12. Symmetric Property	13. Substitution Property					
, so 45 = r	$m \overline{BC}$ . $m \angle ABC = 15x$ , and $x = 2$ .					
	m∠ ABC =					
14. Transitive Property	15. Reflexive Property					
If m $\angle 1 - m \angle 2 = 15^{\circ}$ , and 15° then, = m $\angle 3$ .	=, 10 =					
<u> </u>	·/					

13

\_\_\_\_



### Week: 1 - Day 3

28-30.  mBC = x + 13	Mathematical Step	Reasoning Used
m CD = 3x - 21 $m BD = 60$	$m\overline{BC} + m\overline{CD} = m\overline{BD}$	Segment Addition POST
The figure		Substitution PTY
V z	4x - 8 = 60	Simplify
B C D	4x = 68	
		Division PTY
1-33. m $\overline{BC} = 8x - 2$	Mathematical Step	Reasoning Used
$m\overline{BD} = 45$	$\overline{mBC} + \overline{mCD} = \overline{mBD}$	Segment Addition POST
The figure	8x - 2 + 3x + 3 = 45	
v	11x +1 = 45	Simplify
B C D	11x = 44	
	x = 4	
4-37. m $\overline{BC} = x + 3$ m $\overline{CD} = 3x - 9$	Mathematical Step	Reasoning Used
$m\overline{BD} = 50$	$m\overline{BC} + m\overline{CD} = m\overline{BD}$	Segment Addition POST
The figure	x + 3 + 3x - 9 = 50	
Ťу		Simplify
	4x = 56	
¥		Division PTY
3-39. Duplicate the segment be eps we have learned. Use a ru gments for accuracy.	elow. Then, construct a perpendi ller to check the length of the ori	cular bisector using the ginal and duplicated

Week: 1 – Day 1					
1)	F	2)	A		
3)	Н	4)	С		
5)	В	6)	D		
7)	G	8)	E		
9)	2	10)	3		
11)	3	12)	Plane P		
13)	B, C, and F	14)	A, D, and E		
Answers to	o question 15 may vary, but must be a flat surfa	ace in th	ne room. An example is given.		
15)	The floor [A table, a wall, a book, or any other	flat sur	face in the room could also be correct.]		
16)	7 = a				
17)	Line A is congruent to Line C.				
18)	x + 9 = 21				
19)	6x - 30 = 26  OR  6x - (6)(5) = 26				
20)	$\overline{BC}\cong\overline{BC}$				
21)	19	22)	Subtraction PTY		
23)	Addition PTY	24)	3		
25)	Multiplication PTY	26)	Division PTY		
27)	27	28)	Addition PTY		
29)	Subtraction PTY	30)	7		
31)	Division PTY	32)	Multiplication PTY		
33)	5 [13 – 8 = 5]	34)	81° [43 + 38 = 81]		
35)	$6\left[\frac{2}{3}(18) = 12; 18 - 12 = 6\right]$				
36)	37° [2x + 5 + x – 3 = 122°; 3x + 2 = 122°; 3x =	120°; x	x = 40°; ∠FEG = 40 <b>–</b> 3 = 37°]		
For questic	ons 37-38, the drawings are not shown to scale re shown	, but the	e marks needed to complete the constructions		
37)		38)	*		
		,			
	/		¥		
			-		

39) Error: Step 3 Correct:





Answers\_GRDGEO\_W01\_PTYEQ.docx

\_