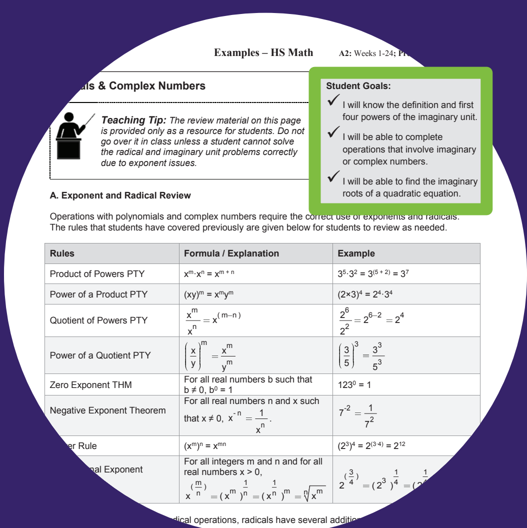
High School Math

Curriculum Sample

A Grade Ahead's rigorous year-round High School Math program covers 6 months of Algebra 2 concepts and 6 months of Pre-Calculus concepts. It is designed to challenge your child to a higher academic standard. Our monthly curriculum includes math concepts and real-world problems to develop strong critical thinking skills. The material also includes one pre-test and one post-test every quarter.

Each week will have an in-depth lesson (which we call Examples), homework (2 days per week), and detailed answers. In these next pages, we offer a closer look at what our examples, homework, and answers offer as well as a specific sample of each.



Examples – HS Math A2: Weeks 1-24;

als & Complex Numbers

Student Goals:

✓ I will know the definition and first four powers of the imaginary unit.

✓ I will be able to complete operations that involve imaginary or complex numbers.

✓ I will be able to find the imaginary roots of a quadratic equation.

Teaching Tip: The review material on this page is provided only as a resource for students. Do not go over it in class unless a student cannot solve the radical and imaginary unit problems correctly due to exponent issues.

A. Exponent and Radical Review

Operations with polynomials and complex numbers require the correct use of exponents and radicals. The rules that students have covered previously are given below for students to review as needed.

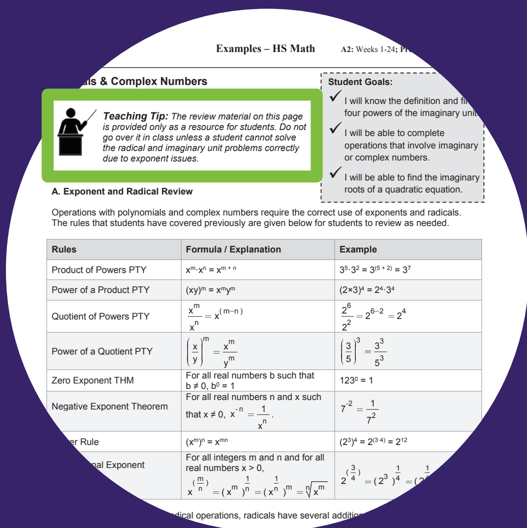
Rules	Formula / Explanation	Example
Product of Powers PTY	$x^m \cdot x^n = x^{m+n}$	$3^5 \cdot 3^2 = 3^{(5+2)} = 3^7$
Power of a Product PTY	$(xy)^m = x^m y^m$	$(2\times3)^4 = 2^4 \cdot 3^4$
Quotient of Powers PTY	$\frac{x^m}{x^n} = x^{(m-n)}$	$\frac{2^6}{2^2} = 2^{6-2} = 2^4$
Power of a Quotient PTY	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3}$
Zero Exponent THM	For all real numbers b such that $b \neq 0$, $b^0 = 1$	$123^0 = 1$
Negative Exponent Theorem	For all real numbers n and x such that $x \neq 0$, $x^{-n} = \frac{1}{x^n}$.	$7^{-2} = \frac{1}{7^2}$
er Rule	$(x^m)^n = x^{mn}$	$(2^3)^4 = 2^{(3\cdot4)} = 2^{12}$
nal Exponent	For all integers m and n and for all real numbers $x > 0$, $x^{\left(\frac{m}{n}\right)} = (x^m)^{\frac{1}{n}} = (x^{\frac{1}{n}})^m = \sqrt[n]{x^m}$	$2^{\left(\frac{3}{4}\right)} = (2^3)^{\frac{1}{4}} = (2$

dical operations, radicals have several additi

Student Goals

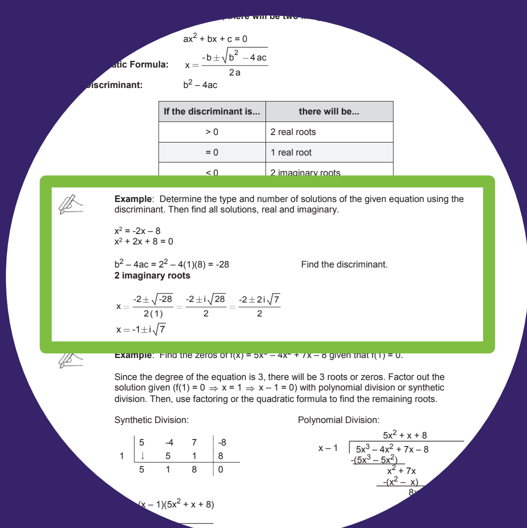
Student goals are listed at the top right of the Examples each week. These are topics that your child should understand by the end of the week.

Lesson pages are titled "Examples – HS Math," answer pages are titled "Answers – HS Math," and homework pages are simply titled "HS Math."



Teaching Tip

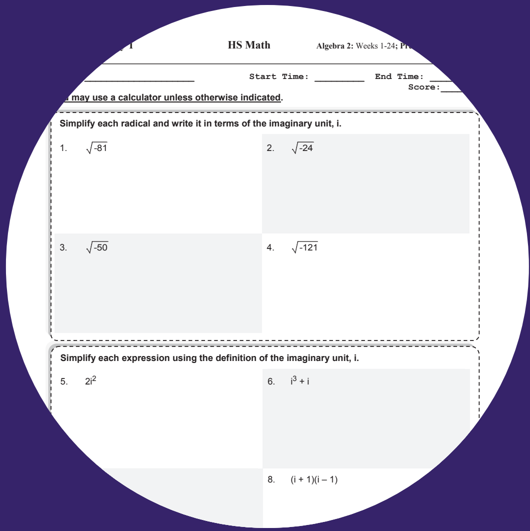
Teaching tips are suggestions to help you or your teacher present the topic to your child. These could include topics to review first or even an activity to do with your child.



Examples

To illustrate the topic, examples are provided to you and your child. These examples help demonstrate how to solve the problem or figure out the answer.

Each day's homework usually takes about 30 minutes to complete.



HS Math **Algebra 2:** Weeks 1-24;

Start Time: ________ End Time: ________
Score: ________

may use a calculator unless otherwise indicated.

Simplify each radical and write it in terms of the imaginary unit, i.

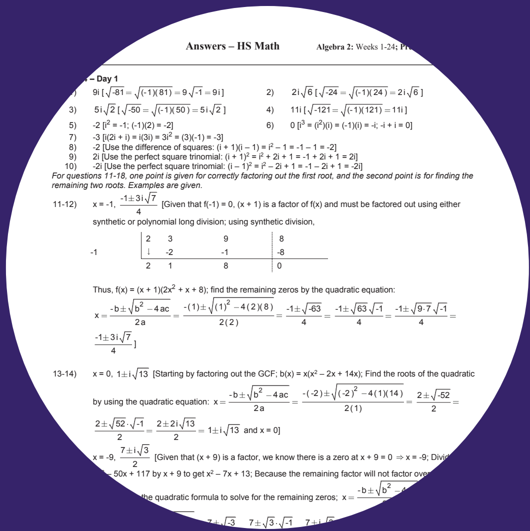
1. $\sqrt{-81}$	2. $\sqrt{-24}$
3. $\sqrt{-50}$	4. $\sqrt{-121}$

Simplify each expression using the definition of the imaginary unit, i.

5. $2i^2$	6. $i^3 + i$
	8. $(i + 1)(i - 1)$

Homework

Each week, two days of homework are given to apply concepts from that week's lesson and reinforce the topic.



Answers – HS Math **Algebra 2:** Weeks 1-24;

– Day 1

2) $2i\sqrt{6}$ [$\sqrt{-24} = \sqrt{(-1)(24)} = 2i\sqrt{6}$]

$9i$ [$\sqrt{-81} = \sqrt{(-1)(81)} = 9\sqrt{-1} = 9i$]

3) $5i\sqrt{2}$ [$\sqrt{-50} = \sqrt{(-1)(50)} = 5i\sqrt{2}$]

4) $11i$ [$\sqrt{-121} = \sqrt{(-1)(121)} = 11i$]

5) -2 [$i^2 = -1$; $(-1)(2) = -2$]

6) 0 [$i^3 = (i^2)(i) = (-1)(i) = -i$; $-i + i = 0$]

7) -3 [$i(2i + i) = i(3i) = 3i^2 = (3)(-1) = -3$]

8) -2 [Use the difference of squares: $(i + 1)(i - 1) = i^2 - 1 = -1 - 1 = -2$]

9) $2i$ [Use the perfect square trinomial: $(i + 1)^2 = i^2 + 2i + 1 = -1 + 2i + 1 = 2i$]

10) $-2i$ [Use the perfect square trinomial: $(i - 1)^2 = i^2 - 2i + 1 = -1 - 2i + 1 = -2i$]

For questions 11-18, one point is given for correctly factoring out the first root, and the second point is for finding the remaining two roots. Examples are given.

11-12) $x = -1, \frac{-1 \pm 3i\sqrt{7}}{4}$ [Given that $f(-1) = 0$, $(x + 1)$ is a factor of $f(x)$ and must be factored out using either synthetic or polynomial long division; using synthetic division,

-1	2	3	9	8
	↓	-2	-1	-8
	2	1	8	0

Thus, $f(x) = (x + 1)(2x^2 + x + 8)$; find the remaining zeros by the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(8)}}{2(2)} = \frac{-1 \pm \sqrt{-63}}{4} = \frac{-1 \pm \sqrt{63}\sqrt{-1}}{4} = \frac{-1 \pm \sqrt{9 \cdot 7}\sqrt{-1}}{4} = \frac{-1 \pm 3i\sqrt{7}}{4}]$$

13-14) $x = 0,\ 1 \pm i\sqrt{13}$ [Starting by factoring out the GCF; $b(x) = x(x^2 - 2x + 14x)$; Find the roots of the quadratic by using the quadratic equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(14)}}{2(1)} = \frac{2 \pm \sqrt{-52}}{2} = \frac{2 \pm \sqrt{52} \cdot \sqrt{-1}}{2} = \frac{2 \pm 2i\sqrt{13}}{2} = 1 \pm i\sqrt{13}$ and $x = 0$]

$x = -9, \frac{7 \pm i\sqrt{3}}{2}$ [Given that $(x + 9)$ is a factor, we know there is a zero at $x + 9 = 0 \Rightarrow x = -9$;

$- 50x + 117$ by $x + 9$ to get $x^2 - 7x + 13$; Because the remaining factor will not factor

the quadratic formula to solve for the remaining zeros;

Answers

Answers are provided to check your child's homework. Enter the scores into the Parent Portal to track progress and note which areas may need more work.

Radicals & Complex Numbers

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Power Rule	$(x^m)^n = x^{mn}$	$(2^3)^4 = 2^{(3\cdot4)} = 2^{12}$
The Rational Exponent Theorem	For all integers m and n and for all real numbers $x > 0$, $x^{\left(\frac{m}{n}\right)} = (x^m)^{\frac{1}{n}} = (x^{\frac{1}{n}})^m = \sqrt[n]{x^m}$	$2^{\left(\frac{3}{4}\right)} = (2^3)^{\frac{1}{4}} = (2^{\frac{1}{4}})^3 = \sqrt[4]{2^3}$

Although exponents include radical operations, radicals have several additional rules, as well.

Taking an even-numbered root of...	Will result in...	Example
a positive number	positive and negative solutions	$\sqrt{x^2} = \pm x$; $\sqrt{9} = \pm 3$; $\sqrt[4]{16} = \pm 2$
zero	1 solution (0)	$\sqrt{0} = 0$; $\sqrt[6]{0} = 0$
a negative number	no real solutions	$\sqrt{-2} = \varnothing$; $\sqrt[4]{-16} = \varnothing$; $\sqrt[6]{-729} = \varnothing$

B. Imaginary & Complex Numbers *Day 1 Q1-2 & 5-6*

The Imaginary Unit

Taking a square root of a negative number is not possible using only real numbers; however, there are some math processes that require taking the square root of a negative number. To make that happen, mathematicians use the **imaginary unit**, i.

$$\mathbf{i} = \sqrt{\mathbf{-1}}$$

Even though **i** is not a real number, it follows the same rules as real numbers when it comes to operations (multiplication, division, addition, etc.). Simply treat it as a variable that always has the same value (like π).

Example: $i + i = 2i = 2\sqrt{-1}$

Example: $i - i = 0$

Example: $\frac{14i}{2i} = 7$

Example: $(i)^2 = (\sqrt{-1})^2 = -1$

Example: $\sqrt{-56} = \sqrt{(-1)(56)} = i\sqrt{56} = 2i\sqrt{14}$

The second to last example above shows how it is possible to find the powers of i using regular solving methods; however, it is useful to have the first four memorized.

Powers of i to Know	How to Find Them
$i = \sqrt{-1}$	Definition of i
$i^2 = -1$	$(i)^2 = (\sqrt{-1})^2 = -1$
$i^3 = -i$	$(i)^3 = (i^2)(i) = (-1)(i) = -i$
$i^4 = 1$	$(i)^4 = (i^2)(i^2) = (-1)(-1) = 1$

Complex Numbers *Day 1 Q8-9*

By definition, a complex number is any number that involves the imaginary unit, i. The **standard form** of a complex number is **a + bi** where a and b are real numbers, and b ≠ 0.

With a + bi, if...	the number will become...	Example
a = 0	a pure imaginary number	1.7i; 12i; -2i
b = 0	a real number (not complex or imaginary at all)	12; -201; 0.5
a ≠ 0, and b ≠ 0	a complex number	4 + 3i; 17 – 4i; -3 + 10.8i

Complex Solutions to Quadratic Equations

Day 1 Q11-14; 19-20

Previously, when finding roots to a parabola, if the discriminant was negative, there was no real solution. **Now, when the discriminant is negative, there will be two imaginary roots**.

Standard Form: $ax^2 + bx + c = 0$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: $b^2 - 4ac$

If the discriminant is...	there will be...
> 0	2 real roots
= 0	1 real root
< 0	2 imaginary roots

Example: Determine the type and number of solutions of the given equation using the discriminant. Then find all solutions, real and imaginary.

$x^2 = -2x - 8$
$x^2 + 2x + 8 = 0$

$b^2 - 4ac = 2^2 - 4(1)(8) = -28$ Find the discriminant.
2 imaginary roots

$$x = \frac{-2 \pm \sqrt{-28}}{2(1)} = \frac{-2 \pm i\sqrt{28}}{2} = \frac{-2 \pm 2i\sqrt{7}}{2}$$

$$x = -1 \pm i\sqrt{7}$$

Example: Find the zeros of $f(x) = 5x^3 - 4x^2 + 7x - 8$ given that $f(1) = 0$.

Since the degree of the equation is 3, there will be 3 roots or zeros. Factor out the solution given ($f(1) = 0 \Rightarrow x = 1 \Rightarrow x - 1 = 0$) with polynomial division or synthetic division. Then, use factoring or the quadratic formula to find the remaining roots.

Synthetic Division:

$$\begin{array}{c|ccc:c} & 5 & -4 & 7 & -8 \\ 1 & \downarrow & 5 & 1 & 8 \\ \hline & 5 & 1 & 8 & 0 \end{array}$$

Polynomial Division:

$$\begin{array}{r l} & 5x^2 + x + 8 \\ \cline{2-2} x - 1 & \big) \; 5x^3 - 4x^2 + 7x - 8 \\ & \underline{-(5x^3 - 5x^2)} \\ & \qquad x^2 + 7x \\ & \qquad \underline{-(x^2 - x)} \\ & \qquad\qquad 8x - 8 \\ & \qquad\qquad \underline{-(8x - 8)} \\ & \qquad\qquad\quad 0 \end{array}$$

$f(x) = (x - 1)(5x^2 + x + 8)$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(5)(8)}}{2(5)} = \frac{-1 \pm \sqrt{-159}}{10} = \frac{-1 \pm i\sqrt{159}}{10}$$

$$x = 1, \ \frac{-1 \pm i\sqrt{159}}{10}$$

Note: There are always n roots for a polynomial of degree n. Some roots may be imaginary.

Note: If a + bi is a root of a polynomial with real coefficients, then a – bi is also a root.

Note: The sum/difference of complex numbers is done the same way as if i were a variable. That is, (a + bi) ± (c + di) = (a ± c) + (b ± d)i.

Example: Given the following information, determine the polynomial.

Degree 3
f(11) = 0
f(2 – 5i) = 0

Given that 2 – 5i is a zero, we also know that 2 + 5i is a zero. Therefore,

$$\begin{aligned}
f(x) &= (x - 11)(x - (2 - 5i))(x - (2 + 5i)) \\
&= (x - 11)(x + (-2 + 5i))(x + (-2 - 5i)) \\
&= (x - 11)(x^2 + (x)(-2 - 5i) + (-2 + 5i)(x) + (-2 + 5i)(-2 - 5i)) \\
&= (x - 11)(x^2 - 2x - 5ix - 2x + 5ix + ((-2)(-2) + (-2)(-5i) + (5i)(-2) + (5i)(-5i))) \\
&= (x - 11)(x^2 - 4x + (4 + 10i - 10i + 25)) \\
&= (x - 11)(x^2 - 4x + 29) \\
&= (x)(x^2) + (x)(-4x) + (x)(29) + (-11)(x^2) + (-11)(-4x) + (-11)(29) \\
&= x^3 - 4x^2 + 29x - 11x^2 + 44x - 319
\end{aligned}$$

$$f(x) = x^3 - 15x^2 + 73x - 319$$

Note: Multiple answers are possible for any problem like the one shown above because if we multiply the polynomial by any non-zero constant, c, the zeros will remain the same.

$f(x) = c(x - 11)(x - (2 - 5i))(x - (2 + 5i))$

Setting f(x) = 0 and solving for x will result in the same roots originally given and therefore any multiple of $f(x) = x^3 - 15x^2 + 73x - 319$ is acceptable.

Note: The perfect square trinomial pattern and difference of squares pattern can be used with complex numbers; however, the complex number has to be the same. For example, (x – (2 + 3i))(x + (2 + 3i)) works for the difference of squares, but (x – (2 + 3i))(x + (2 – 3i)) does not.

Date: ____________________ Start Time: _________ End Time: _________
Score:____/21

You may use a calculator unless otherwise indicated.

Simplify each radical and write it in terms of the imaginary unit, i.

1. $\sqrt{-81}$

2. $\sqrt{-24}$

3. $\sqrt{-50}$

4. $\sqrt{-121}$

Simplify each expression using the definition of the imaginary unit, i.

5. $2i^2$

6. $i^3 + i$

7. $i(2i + i)$

8. $(i + 1)(i – 1)$

9. $(i + 1)(i + 1)$

10. $(i – 1)(i – 1)$

Find the roots, real and imaginary, using synthetic division if needed.

11-12. $f(x) = 2x^3 + 3x^2 + 9x + 8$; $f(-1) = 0$

13-14. $b(x) = x^3 - 2x^2 + 14x$

15-16. $f(x) = x^3 + 2x^2 - 50x + 117$; $(x + 9)$

17-18. $f(x) = 2x^3 - 15x^2 + 34x - 21$; $(x - 1)$

Determine the polynomial with the information given. More than one answer is possible.

19. Degree 3
$f(1 - 2i) = 0$
$f(3) = 0$

20. Degree 4
$h(0) = 0$
$h(i) = 0$
$h(-3) = 0$

21. Degree 3
$f(2 - i) = 0$
$f(7) = 0$

Week: 14 – Day 1

1) $9i$ [$\sqrt{-81}=\sqrt{(-1)(81)}=9\sqrt{-1}=9i$]

2) $2i\sqrt{6}$ [$\sqrt{-24}=\sqrt{(-1)(24)}=2i\sqrt{6}$]

3) $5i\sqrt{2}$ [$\sqrt{-50}=\sqrt{(-1)(50)}=5i\sqrt{2}$]

4) $11i$ [$\sqrt{-121}=\sqrt{(-1)(121)}=11i$]

5) -2 [$i^2 = -1$; $(-1)(2) = -2$]

6) 0 [$i^3 = (i^2)(i) = (-1)(i) = -i$; $-i + i = 0$]

7) -3 [$i(2i + i) = i(3i) = 3i^2 = (3)(-1) = -3$]

8) -2 [Use the difference of squares: $(i + 1)(i – 1) = i^2 – 1 = -1 – 1 = -2$]

9) 2i [Use the perfect square trinomial: $(i + 1)^2 = i^2 + 2i + 1 = -1 + 2i + 1 = 2i$]

10) -2i [Use the perfect square trinomial: $(i – 1)^2 = i^2 – 2i + 1 = -1 – 2i + 1 = -2i$]

For questions 11-18, one point is given for correctly factoring out the first root, and the second point is for finding the remaining two roots. Examples are given.

11-12) $x = -1,\ \dfrac{-1\pm 3i\sqrt{7}}{4}$ [Given that $f(-1) = 0$, $(x + 1)$ is a factor of $f(x)$ and must be factored out using either synthetic or polynomial long division; using synthetic division,

-1	2	3	9	8
	↓	-2	-1	-8
	2	1	8	0

Thus, $f(x) = (x + 1)(2x^2 + x + 8)$; find the remaining zeros by the quadratic equation:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(1)\pm\sqrt{(1)^2-4(2)(8)}}{2(2)}=\frac{-1\pm\sqrt{-63}}{4}=\frac{-1\pm\sqrt{63}\sqrt{-1}}{4}=\frac{-1\pm\sqrt{9\cdot 7}\sqrt{-1}}{4}=$$

$$\frac{-1\pm 3i\sqrt{7}}{4}$$]

13-14) $x = 0,\ 1\pm i\sqrt{13}$ [Starting by factoring out the GCF: $b(x) = x(x^2 – 2x + 14x)$; Find the roots of the quadratic by using the quadratic equation: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(14)}}{2(1)}=\dfrac{2\pm\sqrt{-52}}{2}=$

$$\frac{2\pm\sqrt{52}\cdot\sqrt{-1}}{2}=\frac{2\pm 2i\sqrt{13}}{2}=1\pm i\sqrt{13}$$ and $x = 0$]

15-16) $x = -9,\ \dfrac{7\pm i\sqrt{3}}{2}$ [Given that $(x + 9)$ is a factor, we know there is a zero at $x + 9 = 0 \Rightarrow x = -9$; Divide $x^3 + 2x^2 – 50x + 117$ by $x + 9$ to get $x^2 – 7x + 13$; Because the remaining factor will not factor over the integers, we must use the quadratic formula to solve for the remaining zeros; $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=$

$$\frac{-(-7)\pm\sqrt{(-7)^2-4(1)(13)}}{2(1)}=\frac{7\pm\sqrt{-3}}{2}=\frac{7\pm\sqrt{3}\cdot\sqrt{-1}}{2}=\frac{7\pm i\sqrt{3}}{2}$$]

17-18) $x = 1,\ \dfrac{7}{2},\ 3$ [Given that $(x - 1)$ is a factor, we know there is a zero at $x – 1 = 0$; $x = 1$; Divide $2x^3 – 15x^2 + 34x – 21$ by $x – 1$ to get $2x^2 – 13x + 21$, then $2x^2 – 13x + 21 = (2x – 7)(x – 3)$; Solving for zeros, $2x – 7 = 0$; $x = \dfrac{7}{2}$ and $x – 3 = 0 \Rightarrow x = 3$]

For questions 19-21, students could give the function provided as an answer or any variation where all parts of the equation are multiplied by the same constant. Examples are given.

19) $f(x) = x^3 – 5x^2 + 11x – 15$ [Given $f(1 – 2i) = 0$, we know that $f(1 + 2i)$ is also a root;
$f(x)= (x – (1 – 2i))(x – (1 + 2i))(x – 3) = (x – 1 + 2i)(x – 1 – 2i)(x – 3) = (x^2 – 2x + 5)(x – 3) =$
$x^3 – 5x^2 + 11x – 15$; the general solution is $f(x) = c(x^3 – 5x^2 + 11x – 15)$ where c is a non-zero constant.]

20) $h(x) = x^4 + 3x^3 + x^2 + 3x$ [Given $h(i) = 0$, we know that $h(-i)$ is also a root; $h(x) = x(x - i)(x + i)(x + 3) = x^4 + 3x^3 + x^2 + 3x$; the general solution is $h(x) = c(x^4 + 3x^3 + x^2 + 3x)$ where c is a non-zero constant.]

21) $f(x) = x^3 - 11x^2 + 33x - 35$ [Given that $f(2 - i) = 0$, we know that $f(2 + i)$ is also a root. $f(x) = (x - 7)(x - (2 - i))(x - (2 + i))$; $f(x) = (x - 7)(x^2 - 4x + 5)$; $f(x) = x^3 - 11x^2 + 33x - 35$; The general solution is $f(x) = c(x^3 - 11x^2 + 33x - 35)$ where c is a non-zero constant.]